

My use of AI in mathematical research

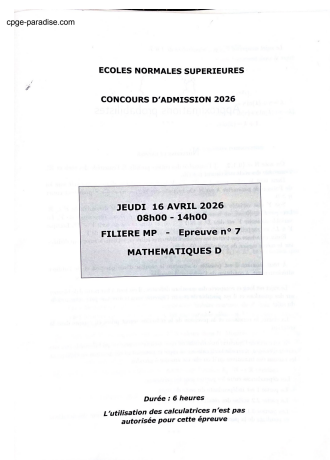
Vincent Boulard

CERMICS & Laboratoire Jacques-Louis Lions

ENS ■ September 2026



What can AI do with this exam?

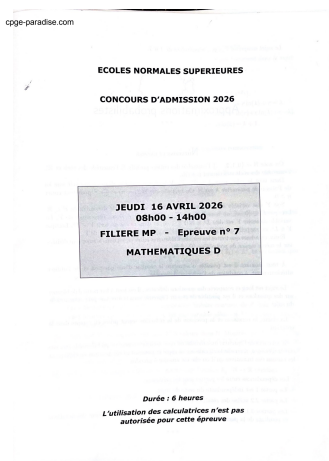


ENS – Math D 2026

A **6-hour** entrance exam.

Probability, analysis,
a sequence of connected arguments.

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The request

A complete set of solutions, using
only the French MP
preparatory-class syllabus.

Solving Math D with GPT-6

My request, paraphrased

Write complete solutions. Include every question with its original numbering. Use only tools from the MP syllabus.

27

numbered questions
covered in the document

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pages of solutions
in $\text{\LaTeX}$ and PDF

$\simeq$ 13 min

to produce the files
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An insufficient assumption in the exam

An insufficient independence assumption in Question 11, identified with an **explicit counterexample**.

Run on 7 September 2026: GPT-6 (Astra), via Codex. [Open the solutions](#)

A counterexample: XOR

Let X_1, X_2 be independent Bernoulli random variables with parameter $1/2$.

$$X_3 = X_1 \oplus X_2 = (X_1 + X_2) \pmod{2}$$

XOR equals 1 exactly when the two bits differ.

X_1	X_2	X_3	Probability
0	0	0	1/4
0	1	1	1/4
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Problem

X_1 is independent of X_2 , and of X_3 .
But it is independent of $X_2 + X_3$?

Pairwise and mutual independence

$$X_2 + X_3 = \begin{cases} 1, & \text{if } X_1 = 1, \\ 0 \text{ or } 2, & \text{if } X_1 = 0. \end{cases}$$

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In Question 11: independence from each variable does not justify the expected cancellation involving their sum.

**The solutions use this counterexample
to identify the missing assumption.**

My research interests

F-equivalence and stabilization of PDEs

Transform a dynamical system to design a stabilizing control.

Sub-Riemannian geometry and hypoelliptic equations

Move in restricted directions; understand how their interactions can recover regularity.

Spectral geometry

Relate the geometry of a space to the eigenvalues and eigenfunctions of an operator.

Mean-field limits and measure differential equations

Pass from many particles to the evolution of a distribution.

I am also starting to explore AI for mathematics.

Source : The four research areas on my homepage.

How I use AI

My experience

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Understand and explore

An example, another approach, a subproblem.

Check

An objection, an assumption, a reference.

Compute and generalize

A known case, then a whole family.

Communicate

A text, a figure, a presentation.

Some examples from my work.

“I don't understand this line of the proof”

Let $A \in \text{GL}_n(\mathbb{R})$, with an induced matrix norm. For $\|H\| \leq (2 \|A^{-1}\|)^{-1}$, a proof states:

The estimate to understand

$$\|(A + H)^{-1} - A^{-1}\| \leq 2 \|A^{-1}\|^2 \|H\|.$$

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$$(A + H)^{-1} - A^{-1} = -(A + H)^{-1}HA^{-1}.$$

An example in PDEs: estimating a difference

My goal: bound a difference

For a nonlinear map $\mathcal{N} : E \rightarrow F$, establish

$$\|\mathcal{N}(u) - \mathcal{N}(v)\|_F \leq C_R \|u - v\|_E \quad \text{if } \|u\|_E, \|v\|_E \leq R.$$

Here, E, F are normed spaces of functions.

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What I can ask

Here is my argument. Where is an estimate missing? Suggest several approaches and explain where each gets stuck.

Models, agents and harnesses

Terminology

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Harness: the software that organizes this work: files, tools, progress tracking and, coordination between agents.

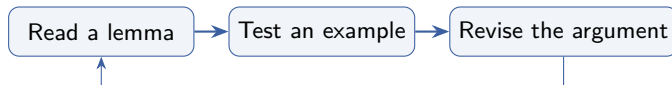
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Procedure	Mathematical question
proof-referee	Which step is unjustified?
conjecture-stress-test	Which example could disprove the claim?
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manuscript-preflight	Are statements, notation and references consistent?

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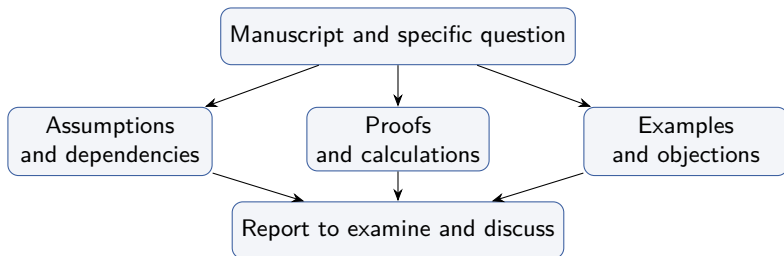
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The expected output

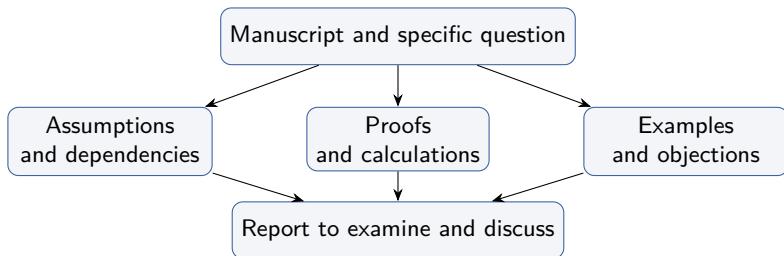
A diagnosis of specific steps: **established** / **unjustified** / **false** / **unresolved**, with reasons.

Numina Studio: checking with multiple agents



One possible organization of a review; schematic overview.

Numina Studio: checking with multiple agents

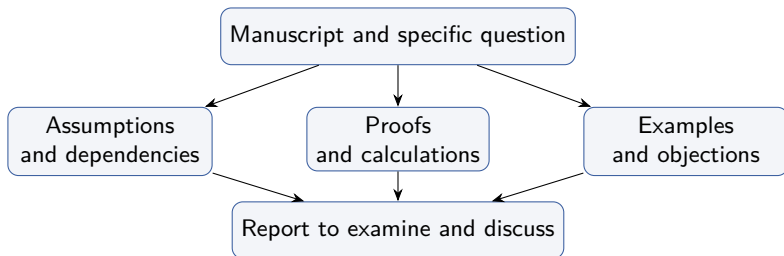


One possible organization of a review; schematic overview.

Semi-formal checking: structured arguments in mathematical language, subjected to checks and objections.

A formal proof is different: a proof object checked by an assistant such as Lean.

Numina Studio: checking with multiple agents



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A formal proof is different: a proof object checked by an assistant such as Lean.

Demo: switch to Numina Studio

Generalizing a calculation: the Grushin example

An example from my research: calculations near a degeneracy curve in a Grushin model.

My starting case

$$X = \partial_x, \quad Y = xe^{x \sin y} \partial_y,$$

$$d\nu = dx dy.$$

$x \in (-\varepsilon, \varepsilon)$, y periodic; $\rho > 0$ weights the volume.

∂_x : differentiate in x . Analytic setting, $\phi(0, y) = 0$.

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The requested generalization

$$X = \partial_x, \quad Y = xe^{\phi(x,y)} \partial_y,$$

$$d\nu = \rho(x, y) dx dy.$$

Purple: the functions being generalized.

Quasimode: an approximate eigenfunction, $Au \simeq \lambda u$.

WKB method: build an approximation using a phase and successive amplitudes.

Allowing a general density

Set $q = x^2 e^{2\phi}$ and $Au = -D_\rho u$, where

$$D_\rho u = \frac{1}{\rho} \partial_x (\rho \partial_x u) + \frac{1}{\rho} \partial_y (\rho q \partial_y u).$$

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Expand using the product rule

$$\begin{aligned} D_\rho u &= u_{xx} + q u_{yy} \\ &\quad + (\partial_x \log \rho) u_x \\ &\quad + q(2\partial_y \phi + \partial_y \log \rho) u_y. \end{aligned}$$

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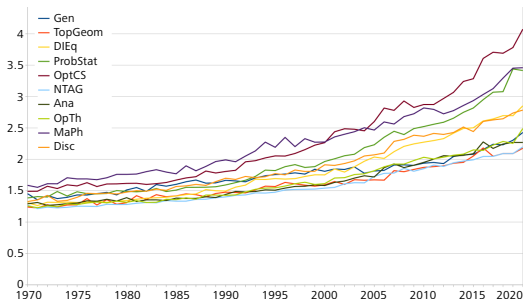
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My request: extend the calculations, recover the case $\rho = 1$, then identify the new equations to solve.

Collaboration in mathematics

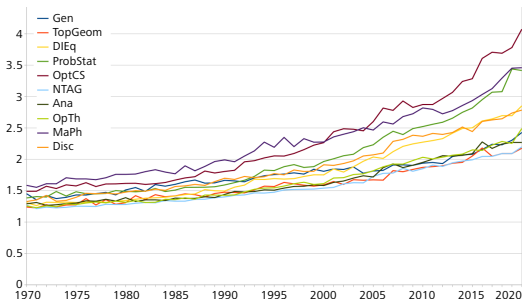
Average number of authors per publication, by field



Original curves, zbMATH Open, 1970–2020. Each colour denotes a field: Ana = analysis; DIEq = differential equations; TopGeom = topology/geometry.

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Original curves, zbMATH Open, 1970–2020. Each colour denotes a field: Ana = analysis; DIFeq = differential equations; TopGeom = topology/geometry.

Coauthorship is increasing across all fields, at different rates.

Source : K. Hulek and O. Teshcke, EMS Magazine (2023), figure 4.

Mathematics and community

William Thurston

“Mathematics only exists in a living community of mathematicians that spreads understanding [...]”

Excerpt from a MathOverflow answer, 2010.

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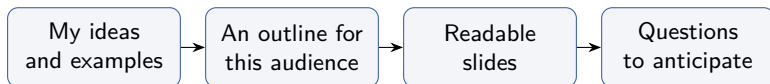
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An idea needs to be explained, discussed, understood and used by others.

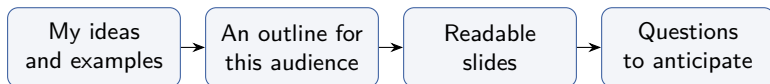
One perspective: if AI speeds up the production of arguments, the quality of discussion and explanation will matter even more.

Source : W. Thurston, What's a mathematician to do?, MathOverflow.

Preparing a scientific presentation

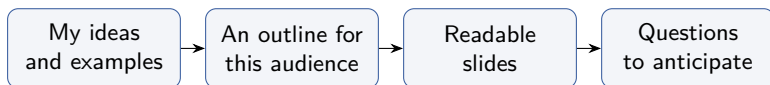


Preparing a scientific presentation



- Choose an example that speaks to you.
- Keep my notation, my style and the meaning of the statements.
- Find what is missing to understand a transition.
- Rehearse: “Why this assumption? What happens in an example?”

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- Rehearse: “Why this assumption? What happens in an example?”

For you: a talk, a student project, group work, explaining something to a friend.

Some example requests

1. Ask for a specific hint

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“Can we choose the same constant on all of $\mathbb{R}$?
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3. Check my understanding

“Here is my proof. Check it, and find if there is an unjustified step.”

Literature searches

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It explores sources, follows leads and produces a report with references you can inspect.

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What I can ask

I know the Weierstrass approximation theorem. How does it extend to continuous functions on a compact space? Explain the assumptions of Stone–Weierstrass, with examples showing why they matter and references to read.

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In my research: find a result with the right assumptions, compare approaches, prepare an annotated bibliography.

Then: open the papers, locate the statements and check how they apply to my problem.

Source : Official Deep Research documentation.

Conclusion and discussion

Four concrete uses

Explore : find approaches.

Check : examine arguments.

Compute : extend a known case.

Communicate : share understanding.

Open questions

How can we learn with these tools?

What new questions could we tackle?

What role for formal proofs?

How will our collaborations change?

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William Thurston

“The product of mathematics is clarity and understanding. Not theorems, by themselves.”

Understanding what a proof explains, choosing questions, making connections and sharing ideas are also part of mathematics.

Source : W. Thurston, What's a mathematician to do?, MathOverflow (2010).

Sources

- Vincent Boulard: research interests and affiliations.
- Math D 2026: original exam scan; local solutions.
Production logs from 7 September 2026; GPT-6 Astra model.
- Numina Studio: public overview.
Jia Li, demo at IHES, 28 May 2026.
- K. Hulek and O. Tschke : How do mathematicians publish?
EMS Magazine (2023), Figure 4; zbMATH Open data.
- Deep Research: official documentation.
- W. Thurston, MathOverflow answer, 2010.
Further reading: On Proof and Progress in Mathematics, 1994.