

# Dirichlet eigenfunctions with nonzero mean for generic domains, and control of the heat equation

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—

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CERMICS & LJLL

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# How do we ever compute a spectrum?

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In practice, we can only compute  $(\lambda_n, \varphi_n)$  when the domain has **symmetries**:

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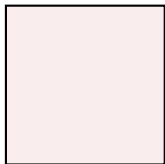
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## The trap

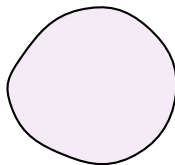
These explicit domains are **exceptional**. Their symmetries produce spectral features (eigenvalue **multiplicities**, rigid nodal patterns) that we then take, wrongly, as the “typical” picture

# Symmetric vs. generic domains



symmetric

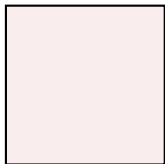
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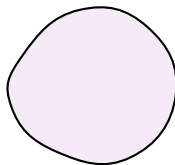
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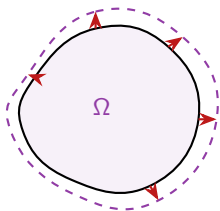
**Idea:** ask what holds for "*almost every*" domain, in the sense of **Baire category**

# A topology on domains (Micheletti)

Fix a reference domain  $D \subset \mathbb{R}^d$  of class  $C^m$ . Deform it by diffeomorphisms:

$$\mathcal{O}^m(D) = \{ \Omega = h(D) : h \in \text{Diff}_0^m(\mathbb{R}^d) \}$$

with the **Courant distance** Micheletti (1972) / Henry: this is a **complete separable metric space**, hence a **Baire space**



A neighbourhood of  $\Omega$  is parametrised by a **single scalar function** on the boundary:

$$\partial\Omega_\sigma = \{ x + \sigma(x) \nu(x) : x \in \partial\Omega \}$$

$$\sigma \in C^m(\partial\Omega)$$

*Deforming a domain  $\leftrightarrow$  choosing  $\sigma$*

# What “generic” means

## Definition (generic property)

*A property is **generic** if it holds on a **residual** set: a countable intersection of open dense sets (a dense  $G_\delta$ ) On a Baire space, residual sets are “large”; their complement is **meager***

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So the multiplicities forced by symmetry are **non-generic**. On the residual set of simple-spectrum domains, each  $\varphi_n$  is determined *up to sign*, questions about  $\varphi_n$  become well posed

(Also generic: eigenfunctions are Morse [Albert]; the squares  $\varphi_n^2$  independent [Privat–Sigalotti]; ...)

# A very poor actuator: scalar control of the heat equation

On  $\Omega$ , drive the heat equation with a **single scalar control** through a *fixed, spatially homogeneous profile* (a **lumped** / Rouchon-type control):

$$\begin{cases} \partial_t z = \Delta z + \textcolor{red}{f}(t) \mathbf{1}_\Omega, & \text{in } \Omega, \\ z = 0 & \text{on } \partial\Omega, \end{cases} \quad \text{or} \quad \begin{cases} \partial_t z = \Delta z, \\ z = \textcolor{red}{c}(t) \text{ on } \partial\Omega \end{cases}$$

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Only a **scalar** function of time; its spatial action is *frozen*

Far weaker than a distributed control  $\mathbf{1}_\omega u(t, x)$

Exact / null controllability are hopeless here  $\Rightarrow$  the natural target is **approximate controllability**

# Where the question comes from: $F$ -equivalence

The question first arose in our study of **rapid stabilization** by  
 **$F$ -equivalence** (Boulard–Hayat 2025)

arXiv:2508.21605

## $F$ -equivalence (feedback equivalence)

Given  $(A, B)$  and a target  $D$  (exp. stable at rate  $\lambda$ ), find an isomorphism  $T$  and a feedback  $K$  with  $T(A + BK) = DT$ ,  $TB = B$ . Then  $A + BK$  is conjugate to  $D \Rightarrow$  **rapid stabilization** at rate  $\lambda$  by  $w = Ku$

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## Parabolic systems (Boulard–Hayat 2025)

For  $A$  parabolic and *finitely many scalar controls*  $B$ , such a  $(T, K)$  exists, and is **unique**  $\iff (A, B)$  is **approximately controllable**

Here  $A = \Delta$ :  $D$  keeps the **eigenbasis**  $\varphi_n$ ; **high modes**  $|\lambda_n| > \lambda$  are *untouched*, the finitely many **low modes**  $|\lambda_n| \leq \lambda$  are **translated past**  $-\lambda \Rightarrow$  finite-dimensional  $K$

# Approximate controllability = nonzero means

Fattorini–Hautus criterion (Badra–Takahashi 2014)

If  $A$  has **simple spectrum**, then  $(A, B)$  is approximately controllable

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(For simplicity we do not specify *in which space* approximate controllability holds, e.g.  $L^2(\Omega)$  for the  $f(t)\mathbf{1}_\Omega$  control, but  $H^{-1}$  in the Rouchon boundary case)

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Everything reduces to one clean condition:

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## Question (Steinerberger–Venkatraman 2025)

*Is it true that, for a **generic** domain, every Dirichlet eigenfunction has **nonzero mean**?*

*They prove a universal lower bound  $\gtrsim (\log n)^{-1/2} n^{1/d}$  on the number of nonzero means, and conjecture the proportion is 1 generically*

# Main result

## Theorem (B. 2026)

Let  $D \subset \mathbb{R}^d$  ( $d \geq 2$ ) be a bounded domain of class  $C^m$ ,  $m \geq 3$ . Then

$$\left\{ \Omega \in \mathcal{O}^m(D) \text{ simple spectrum} : \int_{\Omega} \varphi_n \neq 0 \quad \forall n \geq 1 \right\}$$

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## A word on the literature (to be settled in the paper)

Chitour–Coron–Garavello (2006) already *contain* a proof, not isolated as a statement

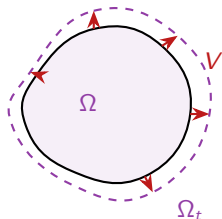
McDonald–Meyers (2003) state it as a theorem, but the transversality step **overlooks a cancellation** on  $\partial\Omega$  (the obstruction below)

This work: **clarify the literature**, give a **robust alternative proof**, and draw the **control-theoretic consequences**

# Shape derivatives: the idea

We must show the bad set  $Z_n = \{\Omega : \int_{\Omega} \varphi_n = 0\}$  has **empty interior**

Deform  $\Omega$  by a **vector field**  $V \in C^m(\overline{\Omega}; \mathbb{R}^d)$ :  $\Omega_t = (\text{id} + t V)(\Omega)$



## Shape derivative

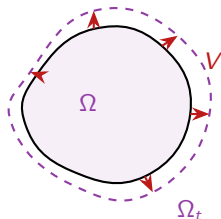
$$dJ(\Omega) \cdot V := \left. \frac{d}{dt} \right|_{t=0} J(\Omega_t), \quad J(\Omega) = \int_{\Omega} \varphi_n$$

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If  $dJ(\Omega) \neq 0$  in **some direction**  $V$ , one can perturb *off*  $Z_n$ : it has **codimension  $\geq 1$**  (empty interior), the analogue of  $\nabla f \neq 0$  on  $\{f = 0\}$

The whole difficulty: can  $dJ(\Omega)$  vanish in *every* direction?

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Differentiate  $J(\Omega_t)$ ,  $\Omega_t = (\text{id} + t V)(\Omega)$ , and read off when  $Z_n$  can fail to be thin

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$$-\Delta\psi - \lambda_n\psi = 1, \quad \psi|_{\partial\Omega} = 0 \quad (\text{solvable} \Leftrightarrow \int_{\Omega} \varphi_n = 0)$$

the derivative becomes a **boundary integral**:

$$\left. \frac{d}{dt} \right|_0 J(\Omega_t) = \int_{\partial\Omega} (V \cdot \nu) \underbrace{\partial_{\nu} \varphi_n \partial_{\nu} \psi}_{=: K_{\Omega}} d\sigma$$

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So the derivative vanishes *in all directions*  $\Leftrightarrow K_{\Omega} \equiv 0$  on  $\partial\Omega$

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**This is the cancellation the classical transversality argument could miss**

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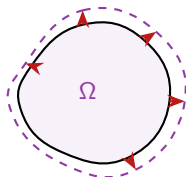
**Degenerate domains sit inside a meager set**

Baire's theorem then closes the argument

# From functions to domains

Why is meagerness of **analytic domains** the same as meagerness of **analytic functions**? Recall the normal parametrisation near  $\Omega$  (for  $\Omega$  with analytic boundary):

$$\partial\Omega_\sigma = \{x + \sigma(x)\nu(x) : x \in \partial\Omega\} \quad \sigma \in C^m(\partial\Omega)$$



## Lemma A.1 (this work)

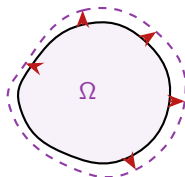
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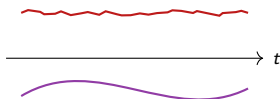
So **meager** analytic  $\sigma \Rightarrow$  **meager** analytic domains It remains to see that  $C^\omega$  is meager in  $C^m$

# Why “too smooth” is rare: the model on an interval

Meagerness of analytic boundaries reduces (in a normal-graph chart) to a statement about the scalar  $\sigma \in C^m(\partial\Omega)$ :

$$C^{m+1}(M) (\supset C^\omega(M)) \text{ is } \textbf{meager} \text{ in } C^m(M)$$

typical  $C^m$ : rough top derivative



$C^{m+1}$ /analytic: exceptional

Same flavour as “**a generic continuous function is nowhere differentiable**”

Mechanism:  $F_n = \{ \partial^m \sigma \text{ } n\text{-Lipschitz} \}$  is **closed, nowhere dense**, and  $C^{m+1} \subset \bigcup_n F_n$

Extra smoothness is a **meager** luxury

## Back to control: a generic positive result

### Corollary (generic controllability & stabilization)

For a **generic** domain  $\Omega$ , the **Rouchon** (uniform boundary) control

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More generally: any  $B$  with  $\langle B, \varphi_n \rangle = c_n \int_{\Omega} \varphi_n$ ,  $c_n \neq 0$  (boundary  $c_n = \lambda_n$ ; interior  $\mathbf{1}_{\Omega}$ ,  $c_n = 1$ )

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**Striking:** Physically counter-intuitive result

# A sharp contrast with Chitour–Coron–Garavello

For the *very same* actuation, a scalar control broadcast along  $\partial\Omega$ , the two results pull in **opposite directions**:

## CCG (2006)

For a generic domain,  
**steady-state controllability**  
**does NOT hold**

## This work

For a generic domain,  
**approximate controllability**  
**HOLDS**

Together they **delineate** exactly which notion of controllability survives on a generic domain

# Summary

## Result

For a **generic** domain, every Dirichlet eigenfunction has **nonzero mean**

**Method:** shape derivative  $\rightarrow$  Schiffer-type obstruction  $\rightarrow$  **bypassed**  
via meagerness of analytic boundaries + Baire

**Control:** generic approximate controllability and rapid stabilization  
of the heat equation by a **single scalar** control, complements CCG

**Thank you!**